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Isotope Effect in Superconducting Cadmium*

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The superconducting transitions of ^{110}Cd, naturally occurring Cd, ^{114}Cd, and ^{116}Cd have been measured. The transition temperatures T_c were found to be 0.5218, 0.5173, 0.5148, and 0.5115 K, respectively, giving $z=0.385\pm0.015$ and $\zeta=0.23$ as parameters in $T_c \propto M^{-z}=M^{-0.5(1-\zeta)}$. These results are compared with various theoretical predictions.

The discovery of the isotope effect in superconductors[1,2] gave great impetus to the development of the microscopic theory by Bardeen, Cooper, and Schrieffer (BCS).[3] More recently, deviations from the BCS isotope effect, i.e., $T_c \propto M^{-1/2}$, have been used to test the predictions of more complex theories of Swihart,[4] Morel and Anderson,[5] Garland,[6] and McMillan.[7] There exists reasonably good agreement between the work of Garland and of McMillan and recent experimental isotope-effect measurements on Zn,[8] Ga,[9] and Cd.[10] This paper presents new measurements on isotopes of Cd which agree qualitatively with the previous result[10] but are of higher precision. This we attribute to our use of larger, more highly purified samples of favorable geometry.

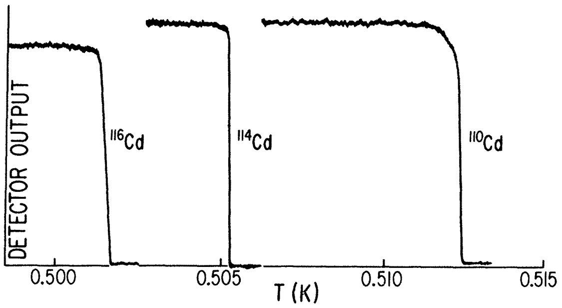

FIG. 1. Superconducting-to-normal transitions of samples of ^{110}Cd, ^{114}Cd, and ^{116}Cd in a longitudinal magnetic field of 1.401 Oe.

EXPERIMENTAL

The apparatus and technique used were similar to those previously described.[8] The Cd samples, as received from Oak Ridge National Laboratory, were not of sufficient purity to give meaningful results. A multiple sublimation and distillation purification similar to that performed[8] on Zn was used. The samples were in the form of right circular cylinders ~3.4 mm in diameter and > 25 mm long. Unlike the Zn, however, the samples were left in the Pyrex tubes following the final crystal growth. The tubes had been lightly coated with carbon so that the metal would not adhere to the glass and be strained during the cooldown. The tubes were open at one end to ensure good thermal contact of the samples with the liquid ^{4}He surrounding them. The reproducibility and sharpness of the transitions indicate sufficient lack of strain and adequate thermal equilibrium for these measurements.

The electrical residual-resistivity ratios of the samples were measured using a modification of the eddy-current-decay measuring circuit of LePage *et al.*[11] The samples all had resistance ratios $r=\rho(4\text{ K})/\rho(300\text{ K})<2.6\times10^{-5}$, $1/r>38\,000$. The isotopic analyses and average atomic masses of the samples are given in Table I.

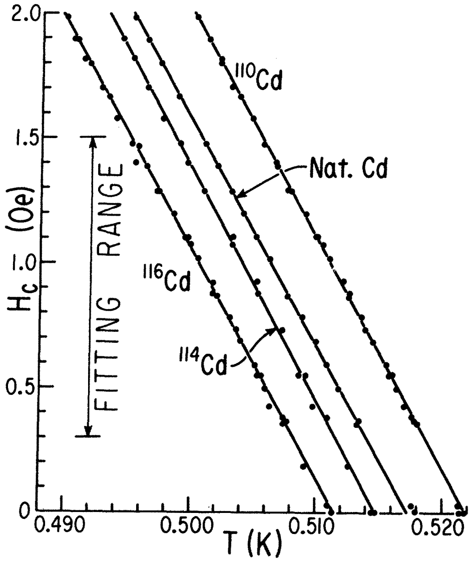

FIG. 2. Critical-field data for samples of Cd. The straight lines are fitted to the data in the range $0.3<H_c<1.5$ Oe.

The nominal 8.2-Ω, $\frac{1}{8}$-W Allen Bradley resistor used as a thermometer was calibrated against the vapor pressure of ^{3}He using the T_{62} ^{3}He scale.[12] Corrections were made for the ^{4}He impurity in the ^{3}He,[13] the thermomolecular pressure ratios,[14] and deviations from the Weber-Schmidt equation due to the use of stainless-steel tubing.[15] The calibration data were least squares fitted to the relation $\ln R=A/T+B+C\ln T$ with a standard deviation of <0.4 mK in every case.

Table I. Isotopic analysis of Cd samples (at. %).

Sample	Cadmium isotope 106	108	110	111	112	113	114	116	Av. at. mass
^{110}Cd	<0.01	<0.01	97.2	1.04	0.90	0.27	0.49	0.09	109.954
Nat.[a]	1.22	0.89	12.43	12.86	23.79	12.34	28.81	7.66	112.424
^{114}Cd	<0.02	<0.02	0.05	0.07	0.23	0.32	99.22	0.12	113.898
^{116}Cd	0.04	0.01	0.20	0.23	0.48	0.32	1.50	97.22	115.818

[a] Handbook values for naturally occurring Cd.

Table II. Superconducting transition temperatures, critical-field slopes at $T=T_c$, and calculated and measured specific-heat jumps at $T=T_c$ for Cd.

Authors	Sample	T_c (K)	$dH_c/dT\,|_{T_c}$ (Oe/K)	ΔC (mJ/mole K)
This work	^{110}Cd	0.5218±0.0002[a]	−91.9	0.44[b]
This work	Nat. Cd	0.5173±0.0001	−91.0	0.43[b]
This work	^{114}Cd	0.5148±0.0003	−93.3	0.45[b]
This work	^{116}Cd	0.5115±0.0002	−91.3	0.43[b]
Palmy[c]	Nat. Cd	0.540		
Phillips[d]	Nat. Cd	0.518		0.470[e]
Brandt *et al.*[f]	Nat. Cd	0.54		
Martin[g]	Nat. Cd	0.52		0.441[e]
Zavaritskiĭ[h]	Nat. Cd	0.53	−95	0.48[b]
Hein and Steele[i]	Nat. Cd	0.555	−99.4	0.55[b]

[a] The error is the standard deviation of the fit to the critical-field data.

[b] Calculated from $dH_c/dT\,|_{T=T_c}$ measurement using the Rutgers formula $\Delta C=T_cV(dH_c/dT)^2|_{T_c}/4\pi$ [see P. Ehrenfest, Commun. Kamerlingh Onnes Lab. Univ. Leiden Suppl. **75b** (1933)]. $V=12.57$ cm^3/mole [C. W. Garland and J. Silverman, Phys. Rev. **119**, 1218 (1960)].

[c] Reference 10.

[d] N. E. Phillips, Phys. Rev. **134**, A385 (1964).

[e] Measured heat-capacity value.

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Table III. Measured and calculated values for z and ζ defined by $T_c\propto M^{-z}=M^{-(1-\zeta)/2}$.

Workers	z	ζ
This work	0.385±0.015[a]	0.23
Palmy[b]	0.32±0.07[a]	0.36
BCS[c]		0.0[d]
Swihart[e]		0.6[d]
Morel and Anderson[f]		0.32[d]
Garland[g]		0.27±0.08[d]
Garland[h]		0.19±0.03[d]
McMillan[i]		0.257[d]

[a] Measured values.

[b] Reference 10.

[c] Reference 3.

[d] Calculated values.

[e] Reference 4.

[f] Reference 5.

[g] Reference 6.

[h] Reference 6 (thesis).

[i] Reference 7. This value was obtained using the pseudopotential calculated by Morel and Anderson (Ref. 5) and ignoring the "strong-coupling" correction $(1+0.62\lambda)/(1+\lambda)$.

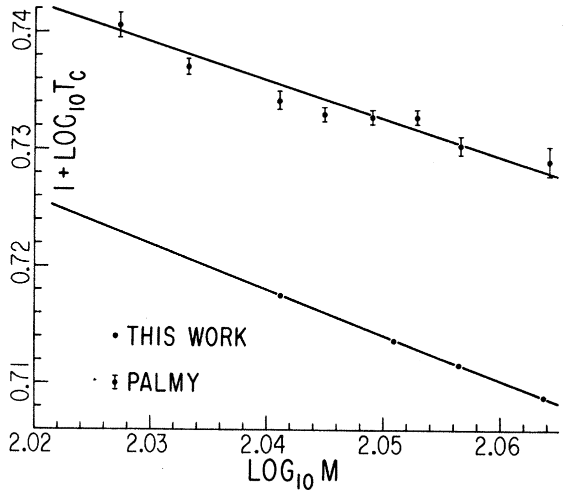

Fig. 3. $1+\log_{10}T_c$ versus $\log_{10}M$ for samples of Cd. The error bars in this work are smaller than the size of the dots.

TABLE IV. Empirical values of the Coulomb pseudopotential μ^* and the electron-phonon coupling constant λ found using the theory of McMillan,[a] and calculated values of μ^*.[b]

Element	T_c (K)	Θ (K)	z	μ_{expt}^*	μ_{theor}^* [b]	λ
Cd[c]	0.5173	209[d]	0.385	0.085	0.09	0.35
Cd[e]	0.5404	209[d]	0.32	0.107	0.09	0.39
Ga[f]	1.0845	325[d]	0.41	0.080		0.36
Zn[g]	0.8471	309[h]	0.37	0.092	0.09	0.37

[a] Reference 7.
[b] Reference 5.
[c] This work.
[d] Reference d of Table II.
[e] Reference 10.
[f] Reference 9.
[g] Reference 8.
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RESULTS AND DISCUSSION

Figure 1 is a tracing of the superconducting-to-normal (S-N) transitions of three of the samples in a longitudinal magnetic field of 1.401 Oe. Figure 2 is the critical-field data over the range used for calculating the isotope effect. All data for three separate cooldowns are shown. The straight lines are least squares fitted to the data in the range $0.3 < H_c < 1.5$ Oe. The transition temperature T_c for each sample is the intersection of the fitted line with $H=0$. The standard deviation of the points from the fitted line was in every case <0.3 mK. In the light of the resistance-ratio measurements and our experience with Zn,[8] no corrections were made in T_c for the presence of impurities. Table II summarizes the values of T_c, $dH_c/dT\,|_{T_c}$, and ΔC (the jump in the heat capacity at T_c) for this work, and other recent measurements.

Figure 3 is a plot of $1+\log_{10}T_c$ versus $\log_{10}M$ (M is the average atomic mass) for the data of this work and that of Palmy.[10] The statistical error in the determination of T_c for this work was less than the size of the dot on the graph. The straight lines are least-squares fits to the data. The standard deviation of the data of this work from the fitted line is 0.06 mK.

Table III compares observed and calculated values of z and ζ for the isotope effect ($T_c \propto M^{-z} = M^{-(1-\zeta)/2}$). The value of z given for these data is that obtained by fitting all data from all runs together, while the error represents the maximum deviation of values of z taken from individual runs from this value.

From the theory of McMillan,[7] the expression for the Coulomb pseudopotential, neglecting the strong-coupling correction, is $\mu^* = (1-2z)^{1/2}\ \ln(\Theta/1.45T_c)$, in which Θ is the Debye temperature. Further, the expression for the electron-phonon coupling constant is

$$\lambda = \frac{1.04+\mu^*\ln(\Theta/1.45T_c)}{(1-0.62\mu^*)\ \ln(\Theta/1.45T_c - 1.04)}\,.$$

We have calculated values of μ^* and λ for Cd, Ga, and Zn using the results of isotope-effect measurements and present the results in Table IV.

In very small magnetic fields, some broadening of the transitions was observed. However, the effect was much smaller than that seen in Zn and Ga.[16] This we attribute to the fact that the isotopic purity of the Cd samples was less than that of those Zn and Ga samples which exhibited the most pronounced excess conductivity above T_c.

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† Now deceased.

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